MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 2, 2017/2018

BSP2014 – INTRODUCTION TO APPLIED PROBABILITY AND STOCHASTIC PROCESSES

(All sections / Groups)

8 MARCH 2018
2.30 p.m. – 4.30 p.m.
(2 Hours)

INSTRUCTIONS TO STUDENTS

1. This question paper consists of **FIVE (5)** printed pages inclusive of the cover page, formulae sheet.

2. Answer **ALL** four questions in the answer booklet provided.

3. Students are allowed to use authorized calculators by lecturer only.

4. Marks are shown at the end of each question.

Question 1 [25 marks]

(a) Automobile warranty claims for engine amount failure in a Troppo Malo 2000 SE are rare at a certain dealership, occurring at a mean rate of 0.1 claim per month.

i) Find the probability of not more than 1 claim in second half of the year. [7 marks]

ii) Find the probability of at least 2 claims in a year. [7 marks]

iii) If the dealer will suffer loss of $100 for every claim, find the expected loss in ten years. [4 marks]

(b) According to Apple's technical support site, downloading a two-hour movie using a 5Mbit/s broadband connection should take 18 to 24 minutes. Assume that the download times are uniformly distributed,

i) what is the probability that the download time will be less than 19 minutes? [4 marks]

ii) what is the mean and variance for the download times? [3 marks]

Question 2 [25 marks]

(a) Given a random variable X has the following probability mass function:

$$f(x)=\begin{cases}\frac{1}{4} & ; \quad x=1,3,5,7 \\ 0 & ; \quad otherwise\end{cases}$$

i) Find the moment generating function [2 marks]

ii) Find the mean and variance for X using the moment generating function. [7 marks]

(b) Random variables X and Y have the following function:

$$f(x,y)=\begin{cases}\frac{1}{2}ye^{-x} & ; \quad x>0; \quad 0<y<2 \\ 0 & ; \quad otherwise\end{cases}$$

i) Determine the marginal pdf for X and Y. [8 marks]

ii) Are X and Y independent? Explain. [3 marks]

iii) Determine the joint CDF for X and Y. [5 marks]

Continued...

Question 3 [25 marks]

(a) Suppose that earthquakes occur in a certain region of California at a rate of five per year.

i) Find the probability that no earthquakes in one year. [3 marks]

ii) Find the probability that at least 2 earthquakes in the first quarter of the year. [6 marks]

iii) Find the probability that the time between earthquakes is less than 6 months. [4 marks]

(b) Penny plays a game with Teresa in which the probability of winning \$1 for Penny is 0.55. Penny starts with \$7 and quits the game when she goes broke or reaches \$15.

i) Find the probability that Teresa goes broke. [6 marks]

ii) Determine the expected duration for the game. [6 marks]

Question 4 [25 marks]

Consider a Markov chain with state space of {1 , 2 , 3} and transition probability matrix as below.

$$\begin{bmatrix} 0.2 & 0.1 & 0.7 \\ 0.1 & 0.9 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

(a) Draw the transition diagram. [3 marks]

(b) Determine whether each of state is absorbing or not. [3 marks]

(c) Find $P(X_2 = 3 \mid X_0 = 2)$. [7 marks]

(d) Find the stationary distribution. [12 marks]

End of Questions

FORMULAE

A. PROBABILITY DISTRIBUTION

Bernoulli Probability Distribution $P(X=x)=p^x q^{1-x}$	for $x=0, 1$
Binomial Probability Distribution $P(X=x)={}^nC_x\, p^x q^{n-x}$	for $x=0, 1, \ldots, n$
Poisson Probability Distribution $P(X=x)=\frac{e^{-\lambda}\lambda^x}{x!}$	for $x=0, 1, 2, \ldots$
Geometric Probability Distribution $P(X=x)=pq^{x-1}$	for $x=1, 2, \ldots$
Uniform Probability Distribution $f(x)=\frac{1}{b-a}$	for $a<x<b$
Exponential Probability Distribution $f(x)=\lambda e^{-\lambda x}$	for $x>0$
Normal Probability Distribution $f(x)=\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$	for $-\infty<x<\infty$

B. MOMENT

k^{th} Moment about the origin of X: $\mu'_k=E[X^k]$
k^{th} Moment about the mean of X: $\mu_k=E[X-\mu]^k$
Moment Generating Function for X: $M_X(t)=E\left(e^{tx}\right)$
Derivative of Moment Generating Function for X: $M_X^{(r)}(t)=E\left(X^r e^{tx}\right)$

C. STOCHASTIC PROCESS

Simple Random Walk:

$$P(X_n = m) = \binom{n}{\frac{n+m}{2}} p^{\left(\frac{n+m}{2}\right)} q^{\left(\frac{n-m}{2}\right)}$$ where m and n are both even or both odd.

$E(X_n) = n(p-q)$; $Var(X_n) = 4npq$

Gambler's Ruin Problem

$$P_a = \begin{cases} \dfrac{\left(\frac{q}{p}\right)^a - \left(\frac{q}{p}\right)^c}{1-\left(\frac{q}{p}\right)^c}, & p \neq q \\ 1-\dfrac{a}{c}, & p = q \end{cases} \quad ; \quad P_a + P_b = 1$$

$$D_a = \begin{cases} \dfrac{1}{q-p}\left\{a - c\left[\dfrac{1-\left(\frac{q}{p}\right)^a}{1-\left(\frac{q}{p}\right)^c}\right]\right\}, & p \neq q \\ a(c-a), & p = q \end{cases}$$

D. POISSON PROCESS

Pmf for N(t) :

$$P\{N(t) = n\} = \frac{e^{-\lambda t}(\lambda t)^n}{n!}$$

Pdf for T_n:

$$f(t) = \lambda e^{-\lambda t}$$

Pdf for S_n:

$$f(t) = \frac{\lambda e^{-\lambda t}(\lambda t)^{n-1}}{(n-1)!}$$